## 2/H–76 (v) (a) (Syllabus–2015)

# 2 0 1 8

( April )

COMMERCE

( Honours )

**( Fundamental Mathematics )**

( BC–202 )

*Marks* : 75

*Time* : 3 hours

*The figures in the margin indicate full marks
for the questions*

Answer *any* **five** questions

1. *(a)* Show that

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc) \qquad 5$$

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(b) If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

compute $A^3 - 23A - 40I$, where $I$ is the identity matrix of order 3.　　5

(c) Firms $A$, $B$ and $C$ supplied 40, 35 and 25 truckloads of stones, and 10, 5 and 8 truckloads of sands to a contractor. If the costs of stones and sand are ₹ 1,200 and ₹ 500 per truck respectively, find the amount paid by the contractor to each of these firms, using matrix method.　　5

2. (a) Define a polynomial function. Under what condition, a polynomial function reduces to a quadratic function?　　2+1=3

(b) Find the domain and the range of
(i) $\sqrt{x^2 - 4}$ and (ii) $\dfrac{1}{|x| - x}$.　　3+3=6

(c) If $f(x) = \log (x + \sqrt{1 + x^2})$, show that
$$f(-x) = -f(x)$$　　3

(d) A truck carrying bricks can travel 100 km/h if it carries less than 120 bricks and travels at a speed of 80 km/h, if it carries 120 or more bricks but less than or equal to 150 bricks. Again, if the truck is overloaded with more than 150 bricks, its speed reduces to 60 km/h. If $x$ is the number of bricks, find the speed function $S(x)$ of the truck.　　3

3. (a) Evaluate the following limits :　　4+3=7

(i) $\displaystyle \lim_{x \to 0} \frac{\sqrt{1 + x} - \sqrt{1 - x}}{x}$

(ii) $\displaystyle \lim_{x \to 0} \frac{(1 + x)^2 - (1 - x)^2}{2x}$

(b) Find the value of $p$ for which
$$f(x) = \begin{cases} p & , \text{ if } x = 5 \\ \frac{x^2 - 5^2}{x - 5} & , \text{ if } x \neq 5 \end{cases}$$
is continuous at $x = 5$.　　4

(c) Draw the graph of the following function : 4

$$f(x) = \begin{cases} \frac{x}{|x|} \,, & \text{if } x < 0 \\ 0 \,, & \text{if } x = 0 \\ \frac{|x|}{x} \,, & \text{if } x > 0 \end{cases}$$

**4.** (a) Find the first-order derivative of the following functions (any *two*) : $4 \times 2 = 8$

(i) $y = x^{\log_e x}$

(ii) $y = \dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$

(iii) $y = (x^2 + 2)^5 (3x^4 - 5)^4$

(b) A TV company has 1000 subscribers who are each paying ₹ 100 per month. The company proposes to increase the monthly subscription and it is believed that for every increase of ₹ 1, five subscribers will discontinue the service. Find what increase in the monthly subscription will yield maximum revenue and what will be the revenue. 7

**5.** (a) A steel plant produces $x$ tons of steel per week at a total cost of

$$₹ \left( \frac{1}{3} x^3 - 5x^2 + \frac{19}{4} x + 35 \right)$$

Find the output level at which the marginal cost attains its minimum. 5

(b) If the demand function is $p = 4 - 5x^2$, find what value of $x$, the elasticity of demand will be unitary, where $p$ is price per unit output and $x$ is the output. 4

(c) If the circular waves in a tank expand so that the circumference increases at a rate of 3 cm/s, find the rate at which the radius of the circle increases. 3

(d) For which values of $x$, the function $x + \dfrac{1}{x}$ has a maximum and a minimum value? 3

**6.** *(a)* A man wants to invest ₹ 5,000 for 4 years. He may invest the amount at 10% per annum compound interest accruing at the end of each quarter of the year or he may invest it at 10·5% per annum compound interest accruing at the end of each year. Which investment will give him better return?     5

*(b)* A machine depreciated in value each year at 10% of its previous value and at the end of the fourth year, its value is ₹ 1,31,220. Find its original value.     5

*(c)* Find the amount of an annuity consisting of payments of ₹ 800 at the end of every 3 months for 3 years at the rate of 8% compounded quarterly.     5

**7.** *(a)* If your expected rate of return on investment is 12% per annum and you find a 10% debenture in the market at ₹ 850, would you like to buy the debenture? Given that the maturity period of the debenture left is 4 years and the maturity value is ₹ 1,000.     5

*(b)* How long will it take for ₹ 10,000 to amount to ₹ 12,000 at 9% interest rate converted monthly?     5

*(c)* A bond presently sells at ₹ 109 which carries a coupon rate of 8 percent per annum. If your expected rate of return is 10 percent, would you like to purchase the bond?     5

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